Gauge coupling and fermion mass relations in low string scale brane models

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Abstract. We analyze the gauge coupling evolution in brane inspired models with $U(3) \times U(2) \times U(1)^N$ symmetry at the string scale. We restrict our work to the case of brane configurations with two and three abelian factors ($N = 2, 3$) and where only one Higgs doublet is coupled to down quarks and leptons and only one to the up quarks. We find that the correct hypercharge assignment of the standard model particles is reproduced for six viable models distinguished by different brane configurations. We investigate the third generation fermion mass relations and find that the correct low energy m_b/m_τ ratio can be obtained for b–τ Yukawa coupling equality at a string scale as low as $M_S \sim 10^3$ TeV.

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1 Introduction

Low scale unification of gauge and gravitational interactions [1–3] appears to be a promising framework for solving the hierarchy problem. In this context, the weakness of the gravitational force at long distances is attributed to the existence of extra dimensions at the Fermi scale. A realization of this scenario can occur in Type I string theory [4] where gauge interactions are mediated by open strings with their ends attached on some D-brane stack, while gravity is mediated by closed strings that propagate in the whole 10-dimensional space.

In the context of Type I string theory using appropriate collections of parallel[1] or intersecting [7,8] D-branes, there has been considerable work in trying to derive the standard model theory or its grand unified extensions [9–18]. Some of these low energy models revealed rather interesting features.
(i) The correct value of the weak mixing angle can be reproduced for a string scale of the order of a few TeV;
(ii) baryon and lepton numbers are conserved due to the existence of exact global symmetries which are remnants of additional anomalous $U(1)$ factors broken by the Green–Schwarz mechanism;
(iii) supersymmetry is not necessary to solve the hierarchy problem.

However, its rivals, supersymmetric grand unified theories and their heterotic string realizations exhibit additional interesting features as:
(i) full gauge coupling unification, that occurs to a scale of the order 10^{16} GeV or a little higher when additional

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[1] For reviews see [5,6].

matter thresholds are introduced (see e.g. [19]), and
(ii) fermion mass relations [20,21] and in particular bottom–tau unification, i.e., the equality of the corresponding Yukawa couplings at the unification scale, which reproduces the correct mass relation m_b/m_τ at low energies.

In Type I string scenarios the volume of the internal space enters in the relation between gauge and string couplings [22] and in general predictability is lost. However, there exist classes of models where (due to some internal volume relation in intersecting brane models or superposition of the associated parallel brane sets) at least the two of the brane couplings are either related or equal at the string scale M_S (e.g., "petite unification" [14,23]). For models with three gauge group factors one such relation is enough to associate the low energy data, i.e., the Weinberg angle and the strong coupling, with the string scale. Given the variety of the Type I vacua one could follow a bottom–up approach and consider ratios of brane couplings as free parameters. In this context, some restrictions on the parameter space can be obtained assuming certain fermion mass relations at the string scale. We will concentrate here on restrictions implied by the heaviest generation fermion mass relations and in particular the bottom–tau Yukawa couplings at the string scale.

Following a bottom–up approach, in this letter we examine the possible brane configurations that can accommodate the standard model and the associated hypercharge embeddings and we analyze the consequences of (partial) gauge coupling unification in conjunction with bottom–tau coupling equality. We shall restrict ourselves to non-supersymmetric configurations, (for some recent results on supersymmetric models see [16]), however, we will consider models with two Higgs doublets so that the bottom and

top quark masses will be related to different vacuum expectation values while the tau lepton and the bottom quark will receive masses from the same Higgs doublet. We find that in a class of models that can be realized in the context of Type I string theory with large extra dimensions, the experimentally low energy masses can be reproduced assuming equality of bottom–tau Yukawa couplings and a string scale as low as $M_S \sim 10^3$ TeV.

In the next section we briefly describe the general set up of brane models and identify candidate brane configurations that allow bottom–tau coupling equality. All possible hypercharge embeddings in the presence of additional $U(1)$ factors are classified. Section 3 deals with the calculational details and renormalization analysis of gauge and Yukawa couplings, while in Sect. 4 the results for b–τ couplings are presented. Our conclusions are drawn in Sect. 5.

2 Hypercharge embedding in generic standard model like brane configurations

In this work, we consider models which arise in the context of various D-brane configurations [9,10]. A single D-brane carries a $U(1)$ gauge symmetry which is the result of the reduction of the ten-dimensional Yang–Mills theory. Therefore, a stack of n parallel, almost coincident D-branes gives rise to a $U(n)$ gauge theory where the gauge bosons correspond to open strings having both their ends attached to some of the branes of the various stacks.

The minimal number of brane sets required to provide the standard model structure is three: a 3-brane "color" stack with gauge symmetry $U(3)_C \sim SU(3)_C \times U(1)$ gives rise to strong interactions, a 2-brane "weak" stack gives rise to $U(2)_L \sim SU(2)_L \times U(1)$ gauge symmetry that includes the weak interactions and an abelian $U(1)$ brane for hypercharge. However, accommodation of all SM particles as open strings between different brane sets requires at least one $U(1)$ brane to be added to the above configuration [9,11]. Additional abelian branes may be present too. In more complicated scenarios the weak or color stacks can be repeated leading to an effective "higher level embedding" of the standard model. The full gauge group will be of the form

$$G = U(m)_C^p \times U(n)_L^q \times U(1)^N, \qquad (1)$$

with $m \geq 3$ and $n \geq 2$ and $p, q \geq 1$. Since $U(n) \sim SU(n) \times U(1)$ and so on, we infer that brane constructions automatically give rise to models with $SU(n)$ gauge group structure and several $U(1)$ factors.

A generic feature of this type of string vacua is that several abelian gauge factors are anomalous. Note that this is in contrast to the heterotic case where at most one $U(1)$ is anomalous, however, anomalies are cancelled by a generalized Green–Schwarz mechanism. At least one $U(1)$ combination remains anomaly free. This is the hypercharge that can be in general written as

$$Y = \sum_{i=1}^{p} k_3^{(i)} Q_3^i + \sum_{j=1}^{q} k_2^{(j)} Q_2^j + \sum_{\ell=1}^{N} k'_\ell Q'_\ell, \qquad (2)$$

where Q_3^i are the $U(1)$ generators of the color factor i, Q_2^j are the $U(1)$ generators of the weak factor j and $Q'_\ell (\ell = 1, \ldots, N)$, are the generators of the remaining abelian factors.

The simplest case which leads directly to the SM theory is the choice $p = q = 1$. Constructions of this type have already been proposed in [9]. An immediate consequence of (1) and (2) is that the hypercharge coupling (g_Y) at the string/brane scale (M_S) is related to the brane couplings (g_m, g_n, g'_i) as[2]

$$\frac{1}{g_Y^2} = \frac{2mk_3^2}{g_m^2} + \frac{2nk_2^2}{g_n^2} + 2\sum_{i=1}^{N} \frac{k_i'^2}{g_i'^2}, \qquad (3)$$

where we have used the traditional normalization $\mathrm{Tr}\, T^a\, T^b = \delta^{ab}/2, a, b = 1, \ldots, n^2$ for the $U(n)$ generators and assumed that the vector representation ($\mathbf{n}$) has an abelian charge $+1$ and thus the $U(1)$ coupling becomes $g_n/\sqrt{2n}$ where g_n the $SU(n)$ coupling.

Choosing further $m = 3, n = 2$ in (3) we obtain directly the non-abelian structure of the SM with several $U(1)$ factors, therefore the hypercharge gauge coupling condition reads

$$\frac{1}{g_Y^2} = \frac{6k_3^2}{g_3^2} + \frac{4k_2^2}{g_2^2} + 2\sum_{i=1}^{N} \frac{k_i'^2}{g_i'^2}. \qquad (4)$$

For a given hypercharge embedding the k_i''s are known and (4) relates the weak angle with the gauge coupling ratios at the string scale

$$\sin^2 \theta_W(M_S) = \frac{1}{1 + k_Y}, \qquad (5)$$

$$k_Y \equiv \frac{\alpha_2}{\alpha_Y} = 6k_3^2 \frac{\alpha_2}{\alpha_3} + 4k_2^2 + 2\sum_{i=1}^{N} k_i'^2 \frac{\alpha_2}{\alpha_i'}, \quad (6)$$

where $\alpha_i \equiv g_i^2/(4\pi)$.

Given a relation between the α_i' and α_2 (or α_3), (5) and (6) in conjunction with the α_3 evolution equation determine the string scale M_S. In the remaining of this section, we will derive all possible sets of k_i's compatible with brane configurations which embed the SM particles and imply an economical Higgs spectrum.

In brane models each SM particle corresponds to an open string stretched between two branes. In our charge conventions, the possible quantum numbers of such a string ending to the $U(m)$ and $U(n)$ brane sets are $(\mathbf{m}; +1, \mathbf{n}; +1)$, $(\overline{\mathbf{m}}; -1, \mathbf{n}; +1)$, $(\mathbf{m}; +1, \overline{\mathbf{n}}; -1)$, $(\overline{\mathbf{m}}; -1, \overline{\mathbf{n}}; -1)$, that is, bifundamentals of the associated unitary groups. Higher antisymmetric or symmetric representations could also be obtained by considering strings with both ends on the same brane set; however, we will restrict here to the bifundamental case. In order to ensure λ_b–λ_τ unification at M_S, we assume models where down quarks and leptons

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[2] If some Cartan generators of $SU(m)$ also contribute to the hypercharge, the formula becomes model dependent; see e.g. [11].
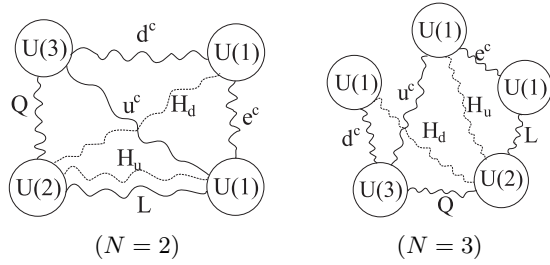
Fig. 1. Possible $N = 2, 3$ brane configurations (N is the number of the $U(1)$ branes) that can accommodate the SM spectrum with down quarks and leptons acquiring masses from the same Higgs doublet. The first consists of four brane sets and has gauge symmetry $U(3) \times U(2) \times U(1)^2$ and the second consists of five brane sets and has gauge group $U(3) \times U(2) \times U(1)^3$

Table 1. Absolute values of the possible hypercharge embedding coefficient sets (k_3, k_2 and k'_i) for the brane configurations with $N = 2$ and $N = 3$ of Fig. 1

N		$|k_3|$	$|k_2|$	$|k'_1|$	$|k'_2|$	$|k'_3|$
	a	$\frac{1}{6}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-$
2	**b**	$\frac{2}{3}$	$\frac{1}{2}$	1	0	$-$
	c	$\frac{1}{3}$	$\frac{1}{2}$	0	1	$-$
	d	$\frac{1}{6}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	e	$\frac{1}{3}$	$\frac{1}{2}$	0	1	1
3	**f**	$\frac{5}{6}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
	g	$\frac{2}{3}$	$\frac{1}{2}$	1	0	0

acquire their masses from a common Higgs. Only two configurations ($N = 2, 3$) share the above properties and are presented pictorially in Fig. 1.

The possible hypercharge embeddings for each configuration can be obtained by solving the hypercharge assignment conditions for SM particles. Generically, the SM particle abelian charges under $U_3(1) \times U_2(1) \times U(1)'_1 \times U(1)'_2$ are of the form $Q(+1, \epsilon_1, 0, 0)$, $d^c(-1, 0, \epsilon_2, 0)$, $u^c(-1, 0, 0, \epsilon_3)$, $L(0, \epsilon_4, 0, \epsilon_5)$, $e^c(0, 0, \epsilon_6, \epsilon_7)$, where $\epsilon_i = \pm 1, i = 1, \ldots, 7$. Then, the SM hypercharge assignment conditions read

$$k_3 + k_2 \epsilon_1 = \frac{1}{6},$$

$$-k_3 + k'_1 \epsilon_2 = \frac{1}{3},$$

$$-k_3 + k \epsilon_3 = -\frac{2}{3}, \tag{7}$$

$$k_2 \epsilon_4 + k'_2 \epsilon_5 = -\frac{1}{2},$$

$$k'_1 \epsilon_6 + k'_2 \epsilon_7 = 1,$$

for Q, d^c, u^c, L and e^c respectively. Here we have used a compact notation where $k = k'_2$ for the first configuration and $k = k'_3$ for the second one[3].

As seen by (2) and (3), only the absolute values of the hypercharge embedding coefficients k_i, k'_i enter the coupling relation at M_S. Solving (7), for the SM particle charges in configuration ($N = 2$) we obtain three possible solutions. These correspond to the (absolute) values for the coefficients presented in cases **a**, **b**, and **c** of Table 1. Configuration $N = 3$ leads to four additional cases, namely **d**, **e**, **f**, and **g** of the same table. If in a particular solution a coefficient k_i (or k'_i) turns out to be zero, the associated abelian factor does not participate in the hypercharge.

3 Gauge coupling running and the string scale

As already mentioned, in the low energy Type I string scenarios the gauge couplings do not unify at the string scale M_S[4]. However, it has been observed that in some cases low energy data are compatible with a partially unified model where some of the gauge couplings are equal ("petite unification" [14, 23]). At the string level this scenario corresponds to superposing the associated parallel brane stacks. Moreover certain coupling relations arise in classes of intersecting brane models. Given the fact that there may exist various gauge coupling relations at the string scale M_S (although only one for a minimal model), low energy electroweak data can be used to determine M_S through the renormalization group equations (RGEs). Following this bottom–up approach, in this section we determine the range of the string scale for all the above models by taking into account the experimental values of $\alpha_3, \alpha_{\text{em}}$ and $\sin^2 \theta_W$ at M_Z [24]

$$\alpha_3 = 0.118 \pm 0.003, \ \alpha_e^{-1} = 127.906, \ \sin^2 \theta_W = 0.23120.$$

For the scales above M_Z we consider the standard model spectrum with two Higgs doublets. The one-loop RGEs for the gauge couplings ($\tilde{\alpha} \equiv \alpha/(4\pi)$) take the form

$$\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad i = Y, 2, 3, \tag{8}$$

where $(b_Y, b_2, b_3) = (7, -3, -7)$ and $t = 2 \ln \mu$ (μ is the renormalization point).

First, we concentrate on simple relations of the gauge couplings, i.e., those relations implied from models arising only in the context on non-intersecting branes. In these cases, certain constraints on the initial values of the gauge couplings have to be taken into account, leading to a discrete number of admissible cases, which we are going to discuss. Thus, in the case of two D5 branes, $U(3)$ and $U(2)$ are confined in different bulk directions. In the parallel brane scenario the orientation of a number of the extra

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[3] In some of our solutions there exist additional unbroken $U(1)$ factors. We will assume in the sequel that these $U(1)$'s will be broken by vacuum expectation values of additional SM singlet Higgs fields or at the string level by six-dimensional anomalies [11].

[4] One may think that the predictability of these constructions is lost, however, bearing in mind that the origin of each gauge factor is due to a different stack of branes, the situation of unequal couplings at the effective string scale, although not predictive, looks completely natural.

Table 2. The possible values of k_Y as a function of $\xi = \frac{\alpha_2}{\alpha_3}$ for various orientations of $U(1)$'s for the models of Table 1. The first row presents the k_Y values when all $U(1)$ branes are aligned with the $SU(2)$ brane, i.e., when $\alpha_i' = \alpha_2$, while the second one corresponds to $\alpha_i' = \alpha_3$. The next two rows show the k_Y values for other possible orientations (see text for details). The last row shows the minimum value of the string scale M_S obtained for the models **a**–**g**

coupling relation	Model					
	a	**b**	**c**	**d**	**e**	**g**
$\alpha_i' = \alpha_2$	$\frac{\xi}{6} + 1$	$\frac{8\xi}{3} + 3$	$\frac{2\xi}{3} + 3$	$\frac{\xi}{6} + \frac{3}{2}$	$\frac{2\xi}{3} + 5$	$\frac{8\xi}{3} + 3$
$\alpha_i' = \alpha_3$	$\frac{7\xi}{6}$	$\frac{14\xi}{3} + 1$	$\frac{8\xi}{3} + 1$	$\frac{2\xi}{3} + 1$	$\frac{14\xi}{3} + 1$	$\frac{14\xi}{3} + 1$
(see text)	$\frac{2\xi}{3} + \frac{1}{2}$	–	–	$\frac{5\xi}{3}$	$\frac{8\xi}{3} + 3$	–
(see text)	–	–	–	$\frac{7\xi}{6} + \frac{1}{2}$	–	–
M_S (GeV)	6.71×10^{17}	5.78×10^3	1.99×10^6	1.65×10^{14}	5.78×10^3	5.78×10^3

$U(1)$'s may coincide with the $U(3)$-stack direction while the remaining abelian branes are parallel to the $U(2)$ stack. This implies that the corresponding $U(1)$ gauge couplings have the same initial values either with the α_3 or with the α_2 gauge couplings. If we define $\xi = \frac{\alpha_2}{\alpha_3}$ the ratio of the two non-abelian gauge couplings at the string scale, for any distinct case, k_Y takes the form $k_Y = \lambda \xi + \nu$, where λ, ν are calculable coefficients which depend on the specific orientation of the $U(1)$ branes. For example, in model **a** we can have the following possibilities: $\alpha_1' = \alpha_2' = \alpha_2$, $\alpha_1' = \alpha_2' = \alpha_3$ and $\alpha_1' = \alpha_2, \alpha_2' = \alpha_3$ leading to $k_Y = \frac{\xi}{6} + 1$, $\frac{7\xi}{6}$ and $\frac{2\xi}{3} + \frac{1}{2}$ respectively. All cases for the models **a**–**g** are presented in Table 2 and are classified with regard to the hypercharge coefficient k_Y. (The analysis shows that all cases of model **f** lead to unacceptably small string scales, so these are not presented.) Allowing α_3 to take values different from α_2, we find that models **a**, **b**, **c**, **d**, **e**, and **g** of Table 1 predict a string scale in a wide range, from a few TeV up to the Planck mass. The highest value is of the order $M_S \sim 7 \times 10^{17}$ GeV and corresponds to equal couplings $\frac{\alpha_2}{\alpha_3} \equiv \xi = 1$ at M_S. On the other hand, lower unification values of the order of a few TeV assume a gauge coupling ratio $\frac{\alpha_3}{\alpha_2} \approx 2$. In this case the idea of complete gauge coupling unification could be still valid, considering that the SM gauge group arises from the breaking of a gauge symmetry whose non-abelian part is $U(3) \times U(2)^2$, i.e., for the case $p = 1$, $q = 2$ of (1) where the factor of 2 in the gauge coupling ratio is related to the diagonal breaking $U(2) \times U(2) \to U(2)$. The distinct cases with the predictions for the unification scale and other quantities we are interested in, are presented in the columns (2)–(4) of Table 3. The lowest possible unification for the three models **b**, **e**, and **g** corresponds to $k_Y = \frac{14\xi}{3} + 1$, and is $M_S \sim 5.81 \times 10^3$ GeV, for a weak to strong gauge coupling ratio $\xi \sim 0.42$ at M_S. Case **c** predicts the intermediate value $M_S = 2 \times 10^6$ GeV while model **d** gives $M_S \sim 10^{14}$ GeV. Finally, model **a** for $\xi \sim 1$ predicts a unification scale as high as $M_S \sim 6.7 \times 10^{17}$ GeV which is of the order of the heterotic string scale. Interestingly, in this latter case, all gauge couplings are equal at M_S, $\alpha_3 = \alpha_2 = \alpha_i'$, while, as
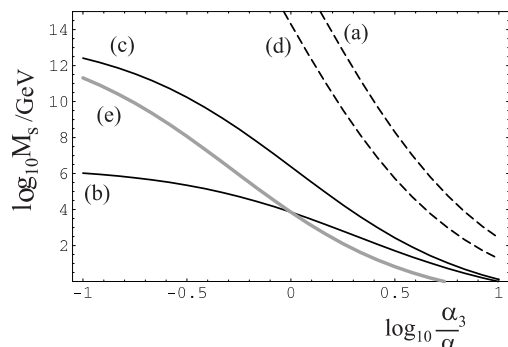
Fig. 2. The string scale as a function of the coupling ratio $\frac{\alpha_3}{\alpha}$ (α is a common value for the $U(1)$ couplings α_i') for the different hypercharge embeddings of Table 1, in the general case of intersecting branes. The results for model **g** are identical with those of model **b**

can be seen from Table 2, k_Y takes a common value for all three cases, $k_Y = 7/6$.

In the general intersecting case, the $U(1)$ branes are neither aligned to the $SU(3)$, nor to the $SU(2)$ stacks; thus the corresponding gauge couplings can take arbitrary values. Without loss of generality, we will assume here for simplicity that all these couplings are equal, $\alpha_1' = \alpha_2' = \ldots = \alpha_N' = \alpha$. In Fig. 2 we plot the string scale (M_S) as a function of the logarithm of the ratio α_3/α for the candidate models **a**, **b**, **c**, **d**, **e**, and **g**. The results for models **b**, **c**, **e**, and **g**, which is identical with model **b**, are represented in the figure with continuous lines. These are compatible with low scale unification particularly when $\alpha_3 \geq \alpha$. For $\alpha_i' = \alpha_3$, (which corresponds to the zero of the logarithm at the x-axis), we obtain again the results of the parallel brane scenario, shown in Table 2. At this point, we further observe a crossing of the **e** curve with the curve for models **b** and **g**. It is exactly for this point ($\alpha_i' = \alpha_3$) that these three models predict the same value for the lowest string scale. When $\alpha_3 \geq \alpha_i'$, model **e** predicts the lowest M_S, whilst, if $\alpha_i' > \alpha_3$, models **b** and **g** imply lower string scales than model **e**.

The values of the string scale for models **a** and **d** (represented in the figure with dashed curves) are substantially

higher; for these latter cases in particular, assuming reasonable gauge coupling relations $\alpha'_i \approx \mathcal{O}(\alpha_{2,3})$ we find that $M_S \geq 10^{12}$ GeV. Again, for $\alpha_3 = \alpha'_i$ (the zero value of the x-axis), we rederive the values of M_S presented in Table 2.

4 Yukawa coupling evolution and mass relations

In this section, we will examine whether unification of the b–τ Yukawa couplings is possible in the above described low string scale models[5]. Our procedure is the following.

Using the experimentally determined values for the third generation fermion masses m_b, m_τ we run the two-loop system of the $SU(3)_C \times U(1)_Y$ renormalization group equations up to the weak scale (M_Z) and reconcile there the results with the experimentally known values for the weak mixing angle and the gauge couplings. For the renormalization group running below M_Z we define the parameters

$$\tilde{\alpha}_e = \left(\frac{e}{4\pi}\right)^2, \quad \tilde{\alpha}_3 = \left(\frac{g_3}{4\pi}\right)^2, \quad t = 2\ln\mu, \qquad (9)$$

where e, g_3 are the electromagnetic and strong couplings respectively and μ is the renormalization scale. The relevant RGEs are [26]

$$\frac{d\tilde{\alpha}_e}{dt} = \frac{80}{9}\tilde{\alpha}_e^2 + \frac{464}{27}\tilde{\alpha}_e^3 + \frac{176}{9}\tilde{\alpha}_e^2\tilde{\alpha}_3,$$

$$\frac{d\tilde{\alpha}_3}{dt} = -\frac{23}{3}\tilde{\alpha}_3^2 - \frac{116}{3}\tilde{\alpha}_3^3 + \frac{22}{9}\tilde{\alpha}_3^2\tilde{\alpha}_e - \frac{9769}{54}\tilde{\alpha}_3^4,$$

$$\frac{dm_b}{dt} = m_b \left\{ -\frac{1}{3}\tilde{\alpha}_e - 4\tilde{\alpha}_3 + \frac{397}{162}\tilde{\alpha}_e^2 - \frac{1012}{18}\tilde{\alpha}_3^2 \right.$$
$$\left. -\frac{4}{9}\tilde{\alpha}_3\tilde{\alpha}_e - 474.8712\tilde{\alpha}_3^3 \right\},$$

$$\frac{dm_\tau}{dt} = m_\tau \left\{ -3\tilde{\alpha}_e + \frac{373}{18}\tilde{\alpha}_e^2 \right\},$$

where m_b, m_τ are the running masses of the bottom quark and the tau lepton respectively, while we use the notation $\tilde{a}_i \equiv g_i^2/16\pi^2$ and $\tilde{a}_{t,b,\tau} \equiv \lambda_{t,b,\tau}^2/16\pi^2$.

The required value for the running mass of m_t at M_Z is computed as follows: we formally solve the one-loop RGE system for $(\tilde{a}_3, \tilde{a}_2, \tilde{a}_Y, \tilde{a}_t, \tilde{a}_b, \tilde{a}_\tau)$ and afterwards we determine the interpolating function for $\tilde{a}_3(\mu)$ and $m_t(\mu; m_t(M_Z))$ at any scale μ above M_Z, where $m_t(M_Z)$ indicates the dependence on an arbitrary initial condition. The unknown value for $m_t(M_Z)$ is determined by solving numerically the algebraic equation

$$\left[m_t\left(\mu; m_t(M_Z)\right) - \frac{M_t}{1 + \frac{16}{3}\tilde{a}_3(\mu) - 2\tilde{a}_t(\mu)} \right]_{\mu=M_t} = 0.$$

We use these results as inputs for the relevant parameters and we run the RGE system to the scale where the $\tilde{a}_b$ and

Table 3. The string scale and the ratio $\xi = \frac{\alpha_2}{\alpha_3}$ for various orientations of $U(1)$ branes presented in Table 2. The last column shows the b–τ ratio at M_S. Exact b–τ unification is obtained in model **c** for $M_S \sim 10^3$ TeV

Model	k_Y	$\xi = \frac{\alpha_2}{\alpha_3}$	M_S/GeV	$\frac{m_b}{m_\tau}(M_S)$
b, e, g	2.969	0.42	5.786×10^3	1.25
c	2.539	0.58	1.986×10^6	1.01
d	1.554	0.93	1.645×10^{14}	0.73
a	1.226	1.01	6.710×10^{17}	0.68

$\tilde{a}_\tau$ Yukawa couplings coincide. In our numerical analysis we use for the gauge couplings the values presented in the previous section, for the bottom quark mass m_b the experimentally determined range at the scale $\mu = m_b$, i.e., $m_b(m_b) = 4.25 \pm 0.15$ GeV and finally the top pole mass is taken to be $M_t = 178.0 \pm 4.3$ GeV [24].

For the scales above M_Z we consider the standard model spectrum augmented by one more Higgs. The Higgs doubling is in accordance with the situation that usually arises in the SM variants with brane origin. Moreover, we assume that one Higgs H_u only couples to the top quark while the second Higgs H_d couples only to the bottom. Then, in analogy with supersymmetry we define the angle β related to their VEVs where $\tan\beta = \frac{v_u}{v_d}$. Thus, we have the equations for the gauge couplings

$$\frac{d\tilde{\alpha}_Y}{dt} = 7\tilde{\alpha}_Y^2, \quad \frac{d\tilde{\alpha}_2}{dt} = -3\tilde{\alpha}_2^2, \quad \frac{d\tilde{\alpha}_3}{dt} = -7\tilde{\alpha}_3^2,$$

and for the Yukawas

$$\frac{d\tilde{\alpha}_t}{dt} = \tilde{\alpha}_t \left(-\frac{17}{12}\tilde{\alpha}_Y - \frac{9}{4}\tilde{\alpha}_2 - 8\tilde{\alpha}_3 + \frac{9}{2}\tilde{\alpha}_t + \frac{1}{2}\tilde{\alpha}_b \right),$$

$$\frac{d\tilde{\alpha}_b}{dt} = \tilde{\alpha}_b \left(-\frac{5}{12}\tilde{\alpha}_Y - \frac{9}{4}\tilde{\alpha}_2 - 8\tilde{\alpha}_3 + \frac{1}{2}\tilde{\alpha}_t + \frac{9}{2}\tilde{\alpha}_b + \tilde{\alpha}_\tau \right),$$

$$\frac{d\tilde{\alpha}_\tau}{dt} = \tilde{\alpha}_\tau \left(-\frac{15}{4}\tilde{\alpha}_Y - \frac{9}{4}\tilde{\alpha}_2 + 3\tilde{\alpha}_b + \frac{5}{2}\tilde{\alpha}_\tau \right),$$

$$\frac{dv_u}{dt} = \frac{v_u}{2} \left(\frac{3}{4}\tilde{\alpha}_Y + \frac{9}{4}\tilde{\alpha}_2 - 3\tilde{\alpha}_t \right),$$

$$\frac{dv_d}{dt} = \frac{v_d}{2} \left(\frac{3}{4}\tilde{\alpha}_Y + \frac{9}{4}\tilde{\alpha}_2 - 3\tilde{\alpha}_b - \tilde{\alpha}_\tau \right),$$

where $t = 2\ln\mu$.

Further, if we define $v^2 = v_u^2 + v_d^2$, with $v_u = v\sin\beta$, $v_d = v\cos\beta$ and $v \sim 174$ GeV, the Z-boson mass is given by $M_Z^2 = \frac{1}{2}(g_Y^2 + g_2^2)v^2$. The electromagnetic and the strong couplings are defined in the usual way:
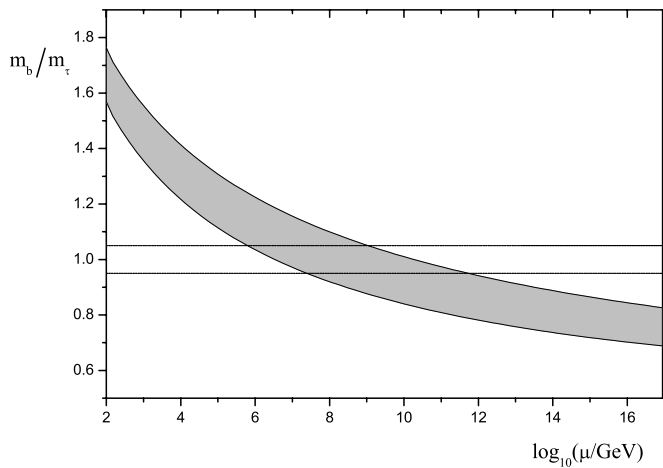
$$\tilde{\alpha}_e = \tilde{\alpha}_Y \cos^2\theta_W = \tilde{\alpha}_2 \sin^2\theta_W,$$

while the top and bottom quark masses are related to the Higgs VEVs by

$$m_t = 4\pi v_u \sqrt{\tilde{\alpha}_t}, \qquad m_b = 4\pi v_d \sqrt{\tilde{\alpha}_b}.$$

We will examine the possibility of obtaining b–τ unification at a low string scale M_S. We first concentrate in the

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[5] For b–τ unification in a different context see also [25].

Fig. 3. The ratio $\frac{m_b}{m_\tau}$ as a function of the energy μ in the two-Higgs standard model. The shaded region corresponds to a_3-uncertainties. The two horizontal lines indicate the interval $\pm 5\%$ around the unity

models **a**–**g** discussed in the previous section. We present our results in the last column of Table 3.

We notice that b–τ unification is obtained in model **c**, for $M_S \approx 2 \times 10^6$ GeV. Models **b**, **e**, and **g** with unification scale $M_S \approx 5.8 \times 10^3$ GeV predict a small (25%) deviation from exact b–τ unification. We observe that in these cases the strong-weak gauge coupling ratio is $a_3 \approx 2\, a_2$. As noted previously, the relation $\alpha_3 = 2\,\alpha_2$ holds naturally if we embed the model in a $U(3) \times U(2)^2 \times U(1)^2$ symmetry.

In Fig. 3 the ratio $\frac{m_b}{m_\tau}$ is plotted as a function of the energy scale for the case of the two-Higgs standard model[6]. All previous uncertainties are incorporated and the result is the shaded region shown in the figure. The horizontal shaded band is defined between the values $\frac{m_b}{m_\tau} = [0.95$–$1.05]$ taking into account deviations of the ratio $\frac{m_b}{m_\tau}$ from unity due to possible threshold as well as mixing effects in the full 3×3 quark and lepton flavor mass matrices. As can be seen, exact $m_b = m_\tau$ equality is found around the scale $M_S \approx 2 \times 10^6$ GeV. Taking into consideration m_b/m_τ-uncertainties expressed through the shaded band, the M_S energy range is extended up to $\sim 10^{12}$ GeV.

5 Conclusions

In this letter, we performed a systematic study of the standard model embedding in brane configurations with $U(3) \times U(2) \times U(1)^N$ gauge symmetry and we examined a number of interesting phenomenological issues. Looking for models with economical Higgs sector, we identified two brane configurations with two or three $(N = 2, 3)$ $U(1)$ branes which can accommodate the standard model where only one Higgs doublet couples to the up quarks, and a second one couples to the down quarks and leptons. We analyzed the possible hypercharge embeddings and found seven possible solutions leading to six models (with acceptable

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[6] For recent work on the two-Higgs model see [27] and references therein.

string scale M_S), implying the correct charge assignments for all standard model particles.

We further examined the gauge coupling evolution in these models for both parallel as well as intersecting branes and determined the lowest string scale allowed for all possible alignments of the $U(1)$ branes with respect to the $U(3)$ and $U(2)$ non-abelian factors of the gauge symmetry. In the parallel brane scenario, we have identified three models which allow for a string scale M_S as low as a few TeV, one model with string scale of the order 10^6 GeV and two models with high unification scales. Similar results were obtained for the general case of intersecting branes.

We further analyzed the consequences of the third generation fermion mass relations and in particular the b–τ equality at the string scale on the above models. In the parallel brane scenario, we found that exact b–τ Yukawa unification is obtained only in the model with $M_S \approx 10^3$ TeV, while in the TeV string scale models the m_b/m_τ ratio deviates from unity by 25%. Allowing the $U(1)$ gauge couplings to take arbitrary (perturbative) values, we found that b–τ Yukawa unification is possible for a wide string scale range form 10^6 up to 10^{12} GeV.

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